

课程: 5.3节 分部积分法

题号	一	二	三	四	五	六	七	八	九	十	总分
得分											
阅卷人											

考生注意: 答题时应注明题号, 并按题号顺序答题。

题号	5.3节 P95	答 题 内 容	廖国勇
一 1	$\int x \cos x dx = \int x d \sin x = x \sin x - \int \sin x dx = x \sin x + \cos x + C$		
2	$\int x e^x dx = \int x d e^x = x e^x - \int e^x dx = x e^x - e^x + C$		
3	$\int \log_2 x dx = x \log_2 x - \int x \cdot d \log_2 x = x \log_2 x - \frac{1}{\ln 2} \int dx$ $= x \log_2 x - \frac{1}{\ln 2} x + C$		
4	$\int \arctan x dx = x \arctan x - \int x \frac{1}{1+x^2} dx = x \arctan x - \frac{1}{2} \ln(1+x^2) + C$		
二 1	求不定积分 $\int x^2 e^{-2x} dx$		
	解: $\int x^2 e^{-2x} dx = \int x^2 d(-\frac{1}{2} e^{-2x}) = -\frac{1}{2} x^2 e^{-2x} + \int \frac{1}{2} e^{-2x} \cdot 2x dx$ $= -\frac{1}{2} x^2 e^{-2x} + \int x d(-\frac{1}{2} e^{-2x})$ $= -\frac{1}{2} x^2 e^{-2x} + x \cdot (-\frac{1}{2} e^{-2x}) + \int -\frac{1}{2} e^{-2x} dx$ $= -\frac{1}{2} x^2 e^{-2x} - \frac{1}{2} x e^{-2x} + \frac{1}{4} e^{-2x} + C$ $= -\frac{1}{2} e^{-2x} (x^2 + x + \frac{1}{2}) + C$		

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10 城东站: 8...

2 求不定积分 $\int x \sin^2 x dx$

$$\begin{aligned}
 \text{解: } \int x \sin^2 x dx &= \int x \cdot \frac{1 - \cos 2x}{2} dx = \int \frac{1}{2} x dx - \frac{1}{2} \int x \cos 2x dx \\
 &= \frac{1}{4} x^2 - \frac{1}{4} \int x d \sin 2x = \frac{1}{4} x^2 - \frac{1}{4} x \sin 2x + \frac{1}{4} \int \sin 2x dx \\
 &= \frac{1}{4} x^2 - \frac{1}{4} x \sin 2x - \frac{1}{8} \cos 2x + C
 \end{aligned}$$

3 求不定积分 $\int \frac{\ln^2 x}{x^2} dx$

$$\begin{aligned}
 \text{解: 原式} &= \int \frac{\ln^2 x}{x} dx = \int \frac{1}{x} d\left(\frac{1}{3} \ln^3 x\right) = \frac{1}{3x} \ln^3 x - \int \frac{1}{3} \ln^3 x d\frac{1}{x} \\
 &= \frac{1}{3x} \ln^3 x + \frac{1}{3x^2} \ln^3 x dx
 \end{aligned}$$

$$\begin{aligned}
 \text{原式} &= \int \ln^2 x d\left(-\frac{1}{x}\right) = -\frac{\ln^2 x}{x} + \int \frac{1}{x} d \ln^2 x \\
 &= -\frac{\ln^2 x}{x} + \int \frac{2 \ln x}{x^2} dx = -\frac{\ln^2 x}{x} - \int 2 \ln x d\left(\frac{1}{x}\right) \\
 &= -\frac{\ln^2 x}{x} - \frac{2 \ln x}{x} + \int \frac{1}{x} d(2 \ln x) \\
 &= -\frac{\ln^2 x}{x} - \frac{2 \ln x}{x} + \int \frac{2}{x^2} dx \\
 &= -\frac{\ln^2 x}{x} - \frac{2 \ln x}{x} - \frac{2}{x} + C
 \end{aligned}$$

4 求不定积分 $\int x \operatorname{arccot} x dx$

$$\begin{aligned}
 \text{解: 原式} &= \int \operatorname{arccot} x d\left(\frac{x^2}{2}\right) = \frac{x^2}{2} \operatorname{arccot} x - \int \frac{x^2}{2} \cdot \left(-\frac{1}{1+x^2}\right) dx \\
 &= \frac{x^2}{2} \operatorname{arccot} x + \frac{1}{2} \int \left(\frac{x^2+1}{1+x^2} - \frac{1}{1+x^2}\right) dx \\
 &= \frac{x^2}{2} \operatorname{arccot} x + \frac{1}{2} x - \frac{1}{2} \arctan x + C
 \end{aligned}$$

$$\begin{aligned}
 \text{注 } \int -\frac{1}{2} \frac{1}{1+x^2} dx &= -\frac{1}{2} \arctan x + C_1 \\
 &= \frac{1}{2} \operatorname{arccot} x + C_2
 \end{aligned}$$

题号

5.3节 P96

答题内容

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二 5 求不定积分 $\int e^{2x} \sin 3x dx$

$$\begin{aligned}
 \text{解: } I &= \int e^{2x} \sin 3x dx = \int \sin 3x d \frac{e^{2x}}{2} \\
 &= \frac{e^{2x}}{2} \sin 3x - \int \frac{e^{2x}}{2} 3 \cos 3x dx \\
 &= \frac{e^{2x}}{2} \sin 3x - \int \frac{3}{4} \cos 3x d e^{2x} \\
 &= \frac{e^{2x}}{2} \sin 3x - \frac{3}{4} e^{2x} \cos 3x + \int \frac{3}{4} e^{2x} (-3) \sin 3x dx \\
 &= \frac{e^{2x}}{2} \sin 3x - \frac{3}{4} e^{2x} \cos 3x - \frac{9}{4} \int e^{2x} \sin 3x dx
 \end{aligned}$$

$$\begin{aligned}
 \Rightarrow I &= \frac{1}{1+\frac{9}{4}} \left(\frac{e^{2x}}{2} \sin 3x - \frac{3}{4} e^{2x} \cos 3x \right) + C \\
 &= e^{2x} \left(\frac{4}{13} \sin 3x - \frac{3}{13} \cos 3x \right) + C
 \end{aligned}$$

一般地求 $\int e^{ax} \sin bx dx$ 与 $\int e^{ax} \cos bx dx$, 可设其不定积分为 $ce^{ax} \sin bx + de^{ax} \cos bx + C$, 然后再确定系数 c 和 d

例如上题, 设 $\int e^{2x} \sin 3x dx = ce^{2x} \sin 3x + de^{2x} \cos 3x + C$

显然有 $[ce^{2x} \sin 3x + d \cdot e^{2x} \cos 3x]' = e^{2x} \sin 3x$

$$\begin{aligned}
 \text{即 } c(2e^{2x} \sin 3x + 3e^{2x} \cos 3x) + d(2e^{2x} \cos 3x - 3de^{2x} \sin 3x) \\
 = (2c - 3d)e^{2x} \sin 3x + (3c + 2d)e^{2x} \cos 3x \\
 = e^{2x} \sin 3x
 \end{aligned}$$

$$\Rightarrow \begin{cases} 2c - 3d = 1 \\ 3c + 2d = 0 \end{cases} \Rightarrow \begin{cases} c = \frac{2}{13} \\ d = -\frac{3}{13} \end{cases}$$

另外, 利用欧拉公式 $e^{ix} = \cos x + i \sin x$ 有 $\sin 3x = \frac{e^{3xi} - e^{-3xi}}{2i}$

$$\text{原式} = \int e^{2x} \frac{e^{3xi} - e^{-3xi}}{2i} dx$$

$$= \frac{1}{2i} \int e^{(2+3i)x} dx - \frac{1}{2i} \int e^{(2-3i)x} dx$$

$$= \frac{1}{2i} \cdot \frac{1}{2+3i} e^{(2+3i)x} - \frac{1}{2i} \cdot \frac{1}{2-3i} e^{(2-3i)x} + C_1$$

$$= e^{2x} \left(\frac{2}{13} \sin 3x - \frac{3}{13} \cos 3x \right) + C$$

$$\text{注, } e^{(2+3i)x} = e^{2x} \cdot e^{3ix} = e^{2x} (\cos 3x + i \sin 3x)$$

$$e^{(2-3i)x} = e^{2x} \cdot e^{-3ix} = e^{2x} (\cos 3x - i \sin 3x)$$

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6 求不定积分 $\int (\arcsin x)^2 dx$ 解: 设 $y = \arcsin x$, 则 $x = \sin y$ $dx = \cos y dy$, 于是

$$\text{原式} = \int y^2 \cos y dy = \int y^2 d \sin y$$

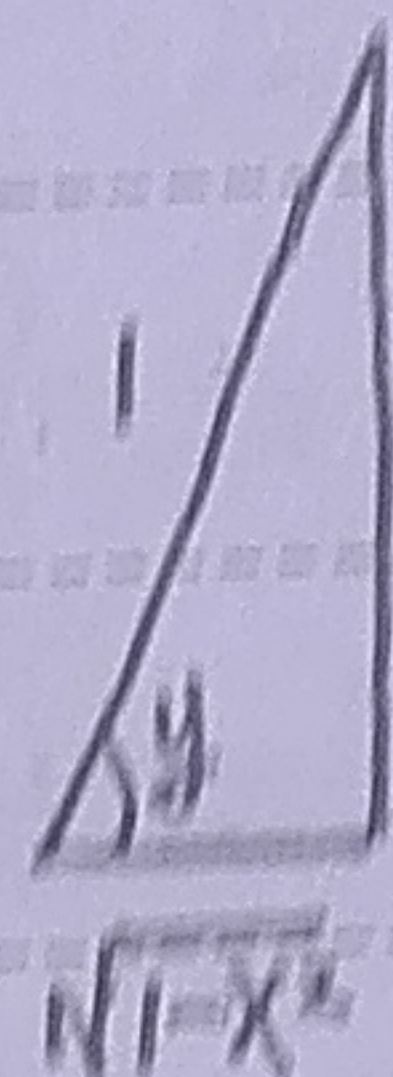
$$= y^2 \sin y - \int \sin y \cdot 2y dy$$

$$= y^2 \sin y + \int 2y d \cos y$$

$$= y^2 \sin y + 2y \cos y - 2 \int \cos y dy$$

$$= y^2 \sin y + 2y \cos y - 2 \sin y + C$$

$$= x(\arcsin x)^2 + 2(\arcsin x) \cdot \sqrt{1-x^2} - 2x + C$$

7 已知 $f'(e^x) = x$, 且 $f(1) = 0$, 求 $f(x)$ 解: 设 $u = e^x$, 则 $x = \ln u$, 于是 $f'(u) = \ln u$, 所以

$$f(x) = \int f'(x) dx = \int \ln x dx = x \ln x - \int x \cdot \frac{1}{x} dx = x \ln x - x + C$$

$$\text{由 } f(1) = 0 \Rightarrow 0 = 1 \times \ln 1 - 1 + C \Rightarrow C = 1, \text{ 即 } f(x) = x \ln x - x + 1$$

8 设 $f(x)$ 的一个原函数为 $\frac{\sin x}{x}$, 求 $\int x f'(x) dx$

$$\text{解: } \int x f'(x) dx = \int x d f(x) = x f(x) - \int f(x) dx$$

$$= x \left(\frac{\sin x}{x} \right)' - \frac{\sin x}{x} + C = x \cdot \frac{x \cos x - \sin x}{x^2} - \frac{\sin x}{x} + C$$

$$= \cos x - \frac{2}{x} \sin x + C$$

注, $f(x)$ 的一个原函数为 $\frac{\sin x}{x}$, 则有

$$\left(\frac{\sin x}{x} \right)' = f(x)$$

$$\int f(x) dx = \frac{\sin x}{x} + C$$

题号	5.3节 P98	答 题 内 容	廖国勇
二 9	求不定积分 $\int e^{\sqrt{x}} dx$	解: 设 $t = \sqrt{x}$, 则 $x = t^2$, $dx = 2t dt$ $\begin{aligned} \int e^{\sqrt{x}} dx &= \int e^t 2t dt = \int 2t^2 de^t = 2te^t - \int e^t 2t dt \\ &= 2te^t - \int 2t de^t = 2te^t - 2te^t + \int e^t \cdot 2 dt \\ &= 2te^t - 2te^t + 2e^t + C \\ &= 2x^{\frac{1}{2}} e^{\sqrt{x}} - 2x^{\frac{1}{2}} e^{\sqrt{x}} + 2e^{\sqrt{x}} + C \end{aligned}$	
10	求不定积分 $\int \frac{x e^x}{(1+e^x)^2} dx$	解: 原式 $= \int x d \frac{-1}{1+e^x} = \frac{-x}{1+e^x} + \int \frac{1}{1+e^x} dx$ $\begin{aligned} &= \frac{-x}{1+e^x} + \int \frac{1}{e^x(1+e^x)} de^x \\ &= \frac{-x}{1+e^x} + \int \left(\frac{1}{e^x} - \frac{1}{1+e^x} \right) de^x \\ &= \frac{-x}{1+e^x} + \ln e^x - \ln(1+e^x) + C \\ &= -\frac{x}{1+e^x} + x - \ln(1+e^x) + C \end{aligned}$	
11	求不定积分 $\int \frac{\ln \sin x}{\sin^2 x} dx$	解: 原式 $= \int \ln \sin x d(-\cot x)$ $\begin{aligned} &= -\cot x \ln \sin x + \int \cot x \cdot \frac{\cos x}{\sin x} dx \\ &= -\cot x \ln \sin x + \int \cot^2 x dx \\ &= -\cot x \ln \sin x + \int (\csc^2 x - 1) dx \\ &= -\cot x \ln \sin x - \cot x - x + C \end{aligned}$	

12 求不定积分 $\int \cos(\ln x) dx$

$$\text{解: 原式} = x \cos(\ln x) - \int x [-\sin(\ln x) \cdot \frac{1}{x}] dx$$

$$= x \cos(\ln x) + \int \sin(\ln x) dx$$

$$= x \cos(\ln x) + x \sin(\ln x) - \int x \cdot \cos(\ln x) \cdot \frac{1}{x} dx$$

$$= x \cos(\ln x) + x \sin(\ln x) - \int \cos(\ln x) dx$$

$$\Rightarrow \int \cos(\ln x) dx = \frac{x}{2} [\cos(\ln x) + \sin(\ln x)] + C$$

13 求不定积分 $\int \frac{\arctan x}{x^2(1+x^2)} dx$

$$\text{解: 设 } \theta = \arctan x, \text{ 则 } x = \tan \theta, dx = \sec^2 \theta d\theta$$

$$1+x^2 = 1+\tan^2 \theta = \sec^2 \theta, \text{ 于是}$$

$$\text{原式} = \int \frac{\theta}{\tan \theta \cdot \sec^2 \theta} \cdot \sec^2 \theta d\theta$$

$$= \int (\cot^2 \theta) \cdot \theta d\theta$$

$$= \int (\csc^2 \theta - 1) \theta d\theta$$

$$= \int \theta \csc^2 \theta d\theta - \int \theta d\theta$$

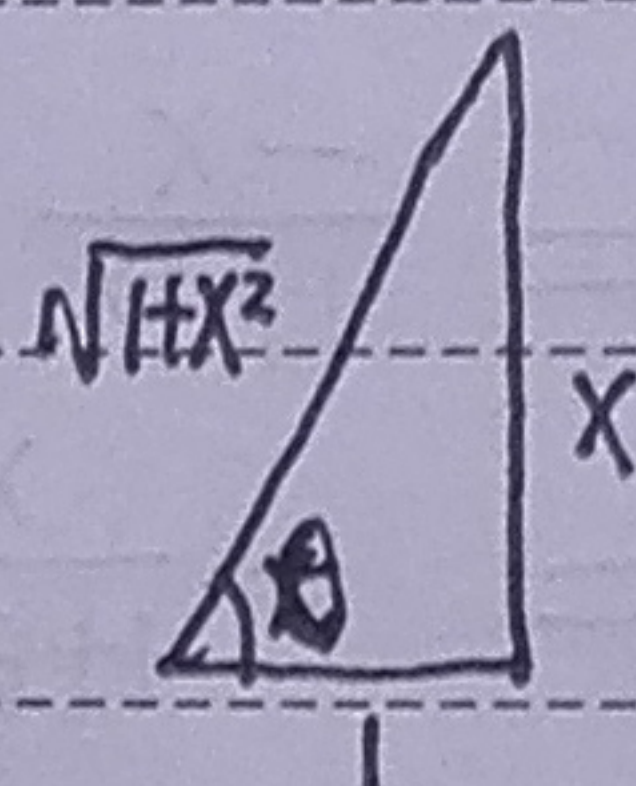
$$= \int \theta d(-\cot \theta) - \frac{1}{2} \theta^2$$

$$= -\theta \cot \theta + \int \cot \theta d\theta - \frac{1}{2} \theta^2$$

$$= -\theta \cot \theta + \ln |\sin \theta| - \frac{1}{2} \theta^2 + C$$

$$= -(\arctan x) \cdot \frac{1}{x} + \ln \left| \frac{x}{\sqrt{1+x^2}} \right| - \frac{1}{2} (\arctan x)^2 + C$$

$$= -\frac{\arctan x}{x} + \ln \left| \frac{x}{\sqrt{1+x^2}} \right| - \frac{1}{2} (\arctan x)^2 + C$$



题号

5.3节 P99

答 题 内 容

廖国勇

二 14 求不定积分 $\int e^x (\frac{1}{x} + \ln x) dx$

$$\begin{aligned}\text{解: 原式} &= \int e^x \frac{1}{x} dx + \int e^x \ln x dx \\ &= \int e^x d \ln x + \int \ln x d e^x \\ &= e^x \ln x + C\end{aligned}$$

注, $\int u(x) dv(x) + \int v(x) du(x) = u(x)v(x) + C$